Riemannian Geometry And Geometric Analysis Universitext

This book introduces advanced undergraduates to Riemannian geometry and mathematical general relativity. The overall strategy of the book is to explain the concept of curvature via the Jacobi equation which, through discussion of tidal forces, further helps motivate the Einstein field equations. After addressing concepts in geometry such as metrics, covariant differentiation, tensor calculus and curvature, the book explains the mathematical framework for both special and general relativity. Relativistic concepts discussed include (initial value formulation of) the Einstein equations, stress-energy tensor, Schwarzschild space-time, ADM mass and geodesic incompleteness. The concluding chapters of the book introduce the reader to geometric analysis: original results of the author and her undergraduate student collaborators illustrate how methods of analysis and differential equations are used in addressing questions from geometry and relativity. The book is mostly self-contained and the reader is only expected to have a solid foundation in multivariable and vector calculus and linear algebra. The material in this book was first developed for the 2013 summer program in geometric analysis at the Park City Math Institute, and was recently modified and expanded to reflect the author's experience of teaching mathematical general relativity to advanced undergraduates at Lewis & Clark College.

This book aims to bridge the gap between probability and differential geometry. It gives two constructions of Brownian motion on a Riemannian manifold: an extrinsic one where the manifold is realized as an embedded submanifold of Euclidean space and an intrinsic one based on the ``rolling'' map. It is then shown how geometric quantities (such as curvature) are reflected by the behavior of Brownian paths and how that behavior can be used to extract information about geometric quantities. Readers should have a strong background in analysis with basic knowledge in stochastic calculus and differential geometry. Professor Stroock is a highly-respected expert in probability and analysis. The clarity and style of his exposition further enhance the quality of this volume. Readers will find an inviting introduction to the study of paths and Brownian motion on Riemannian manifolds.

Offering some of the topics of contemporary mathematical research, this fourth edition includes a systematic introduction to Kahler geometry and the presentation of additional techniques from geometric analysis.

This book provides a comprehensive account of a modern generalisation of differential geometry in which coordinates need not commute. This requires a reinvention of differential geometry that refers only to the coordinate algebra, now possibly noncommutative, rather than to actual points. Such a theory is needed for the geometry of Hopf algebras or quantum groups, which provide key examples, as well as in physics to model quantum gravity effects in the form of quantum spacetime. The mathematical formalism can be applied to any algebra and includes graph geometry and a Lie theory of finite groups. Even the algebra of 2 x 2 matrices turns out to admit a rich moduli of quantum Riemannian geometries. The approach taken is a `bottom up’ one in which the different layers of geometry are built up in succession, starting from differential forms and proceeding up to the notion of a quantum `Levi-Civita’ bimodule connection, geometric Laplacians and, in some cases, Dirac operators. The book also covers elements of Connes’ approach to the subject coming from cyclic cohomology and spectral triples. Other topics include various other cohomology theories, holomorphic structures and noncommutative D-modules. A unique feature of the book is its constructive approach and its wealth of examples drawn from a large body of literature in mathematical physics, now put on a firm algebraic footing. Including exercises with solutions, it can be used as a textbook for advanced courses as well as a reference for researchers.

This book describes very recent results involving an extensive use of analytical tools in the study of geometrical and topological properties of complete Riemannian manifolds. It analyzes in detail an extension of the Bochner technique to the non compact setting, yielding conditions which ensure that solutions of geometrically significant differential equations either are trivial (vanishing results) or give rise to finite dimensional vector spaces (finiteness results). The book develops a range of methods, from spectral theory and qualitative properties of solutions of PDEs, to comparison theorems in Riemannian geometry and potential theory.

Differential Geometry and Symmetric Spaces

Many problems in general relativity are essentially geometric in nature, in the sense that they can be understood in terms of Riemannian geometry and partial differential equations. This book is centered around the study of mass in general relativity using the techniques of geometric analysis. Specifically, it provides a comprehensive treatment of the positive mass theorem and closely related results, such as the Penrose inequality, drawing on a variety of tools used in this area of research, including minimal hypersurfaces, conformal geometry, inverse mean curvature flow, conformal flow, spinors and the Dirac operator, marginally outer trapped surfaces, and density theorems. This is the first time these topics have been gathered into a single place and presented with an advanced graduate student audience in mind; several dozen exercises are also included. The main prerequisite for this book is a working understanding of Riemannian geometry and basic knowledge of elliptic linear partial differential equations, with only minimal prior knowledge of physics required. The second part of the book includes a short crash course on general relativity, which provides background for the study of asymptotically flat initial data sets satisfying the dominant energy condition.

What is the title of this book intended to signify, what connotations is the adjective "Postmodern" meant to carry? A potential reader will surely pose this question. To answer it, I should describe what distinguishes the approach to analysis presented here from what has been called "Modern Analysis" by its protagonists. "Modern Analysis" as represented in the works of the Bour baki group or in the textbooks by Jean Dieudonne is characterized by its systematic and axiomatic treatment and by its drive towards a high level of abstraction. Given the tendency of many prior treatises on analysis to degen erate into a collection of rather unconnected tricks to solve special problems, this definitively represented a healthy achievement. In any case, for the de velopment of a consistent and powerful mathematical theory, it seems to be necessary to concentrate solelyon the internal problems and structures and to neglect the relations to other fields of scientific, even of mathematical study for a certain while. Almost complete isolation may be required to reach the level of intellectual elegance and perfection that only a good mathematical theory can acquire. However, once this level has been reached, it might be useful to open one's eyes again to the inspiration coming from concrete ex ternal problems.

Unlike many other texts on differential geometry, this textbook also offers interesting applications to geometric mechanics and general relativity. The first part is a concise and self-contained introduction to the basics of manifolds, differential forms, metrics and curvature. The second part studies applications to mechanics and relativity including the proofs of the Hawking and Penrose singularity theorems. It can be independently used for one-semester courses in either of these subjects. The main ideas are illustrated and further developed by numerous examples and over 300 exercises. Detailed solutions are provided for many of these exercises, making An Introduction to Riemannian Geometry ideal for self-study.

This volume includes expanded versions of the lectures delivered in the Graduate Minicourse portion of the 2013 Park City Mathematics Institute session on Geometric Analysis. The papers give excellent high-level introductions, suitable for graduate students wishing to enter the field and experienced researchers alike, to a range of the most important areas of geometric analysis. These include: the general issue of geometric evolution, with more detailed lectures on Ricci flow and Kähler-Ricci flow, new progress on the analytic aspects of the Willmore equation as well as an introduction to the recent proof of the Willmore conjecture and new directions in min-max theory for geometric variational problems, the current state of the art regarding minimal surfaces in R3, the role of critical metrics in Riemannian geometry, and the modern perspective on the study of eigenfunctions and eigenvalues for Laplace–Beltrami operators.

Provides a comprehensive and self-contained introduction to sub-Riemannian geometry and its applications. For graduate students and researchers.

Geometric Analysis combines differential equations with differential geometry. An important aspect of geometric analysis is to approach geometric problems by studying differential equations. Besides some known linear differential operators such as the Laplace operator, many differential equations arising from differential geometry are nonlinear. A particularly important example is the Monge-Amperè equation. Applications to geometric problems have also motivated new methods and techniques in differential equations. The field of geometric analysis is broad and has had many striking applications.This handbook of geometric analysis—the first of the two to be published in the ALM series—presents introductions and survey papers treating important topics in geometric analysis, with their applications to related fields. It can be used as a reference by graduate students and by researchers in related areas.

This book contains a clear exposition of two contemporary topics in modern differential geometry: distance geometric analysis on manifolds, in particular, comparison theory for distance functions in spaces which have well defined bounds on their curvature the application of the Lichnerowicz formula for Dirac operators to the study of Gromov's invariants to measure the K-theoretic size of a Riemannian manifold. It is intended for both graduate students and researchers.

Ricci flow is a powerful analytic method for studying the geometry and topology of manifolds. This book is an introduction to Ricci flow for graduate students and mathematicians interested in working in the subject. To this end, the first chapter is a review of the relevant basics of Riemannian geometry. For the benefit of the student, the text includes a number of exercises of varying difficulty. The book also provides brief introductions to some general methods of geometric analysis and other geometric flows. Comparisons are made between the Ricci flow and the linear heat equation, mean curvature flow, and other geometric evolution equations whenever possible. Several topics of Hamilton's program are covered, such as short time existence, Harnack inequalities, Ricci solitons, Perelman's no local collapsing theorem, singularity analysis, and ancient solutions. A major direction in Ricci flow, via Hamilton's and Perelman's works, is the use of Ricci flow as an approach to solving the Poincare conjecture and Thurston's geometrization conjecture.

Book V completes the discussion of the first four books by treating in some detail the analytic results in elliptic operator theory used previously. Chapters 16 and 17 provide a treatment of the techniques in Hilbert space, the Fourier transform, and elliptic operator theory necessary to establish the spectral decomposition theorem of a self-adjoint operator of Laplace type and to prove the Hodge Decomposition Theorem that was stated without proof in Book II. In Chapter 18, we treat the de Rham complex and the Dolbeault complex, and discuss spinors. In Chapter 19, we discuss complex geometry and establish the Kodaira Embedding Theorem.

This book demonstrates the influence of geometry on the qualitative behaviour of solutions of quasilinear PDEs on Riemannian manifolds. Motivated by examples arising, among others, from the theory of submanifolds, the authors study classes of coercive elliptic differential inequalities on domains of a manifold M with very general nonlinearities depending on the variable x, on the solution u and on its gradient. The book highlights the mean curvature operator and its variants, and investigates the validity of strong maximum principles, compact support principles and Liouville type theorems. In particular, it identifies sharp thresholds involving curvatures or volume growth of geodesic balls in M to guarantee the above properties under appropriate Keller-Osserman type conditions, which are investigated in detail throughout the book, and discusses the geometric reasons behind the existence of such thresholds. Further, the book also provides a unified review of recent results in the literature, and creates a bridge with geometry by studying the validity of weak and strong maximum principles at infinity, in the spirit of Omori-Yau's Hessian and Laplacian principles and subsequent improvements.

Basic techniques for researchers interested in the field of geometric analysis.

This established reference work continues to provide its readers with a gateway to some of the most interesting developments in contemporary geometry. It offers insight into a wide range of topics, including fundamental concepts of Riemannian geometry, such as geodesics, connections and curvature; the basic models and tools of geometric analysis, such as harmonic functions, forms, mappings, eigenvalues, the Dirac operator and the heat flow method; as well as the most important variational principles of theoretical physics, such as Yang-Mills, Ginzburg-Landau or the nonlinear sigma model of quantum field theory. The present volume connects all these topics in a systematic geometric framework. At the same time, it equips the reader with the working tools of the field and enables her or him to delve into geometric research. The 7th edition has been systematically reorganized and updated. Almost no page has been left unchanged. It also includes new material, for instance on symplectic geometry, as well as the Bishop-Gromov volume growth theorem which elucidates the geometric role of Ricci curvature. From the reviews:“This book provides a very readable introduction to Riemannian geometry and geometric analysis... With the vast development of the mathematical subject of geometric analysis, the present textbook is most welcome.” Mathematical Reviews “For readers familiar with the basics of differential geometry and some acquaintance with modern analysis, the book is reasonably self-contained. The book succeeds very well in laying out the foundations of modern Riemannian geometry and geometric analysis. It introduces a number of key techniques and provides a representative overview of the field.” Monatshefte für Mathematik

Presents many major differential geometric acheivements in the theory of CR manifolds for the first time in book form Explains how certain results from analysis are employed in CR geometry Many examples and explicitly worked-out proofs of main geometric results in the first section of the book making it suitable as a graduate main course or seminar textbook Provides unproved statements and comments inspiring further study

Riemannian Geometry and Geometric AnalysisSpringer

This text focuses on developing an intimate acquaintance with the geometric meaning of curvature and thereby introduces and demonstrates all the main technical tools needed for a more advanced course on Riemannian manifolds. It covers proving the four most fundamental theorems relating curvature and topology: the Gauss-Bonnet Theorem, the Cartan-Hadamard Theorem, Bonnet’s Theorem, and a special case of the Cartan-Ambrose-Hicks Theorem.

This book deals with such important subjects as variational methods, the continuity method, parabolic equations on fiber bundles, ideas concerning points of concentration, blowing-up technique, geometric and topological methods. It explores important geometric problems that are of interest to many mathematicians and scientists but have only recently been partially solved.

The book gives both invariant global notation and tensor notation and should thus prove to be of interest to physicists as well.

Over the past 15 years, there has been a growing need in the medical image computing community for principled methods to process nonlinear geometric data. Riemannian geometry has emerged as one of the most powerful mathematical and computational frameworks for analyzing such data. Riemannian Geometric Statistics in Medical Image Analysis is a complete reference on statistics on Riemannian manifolds and more general nonlinear spaces with applications in medical image analysis. It provides an introduction to the core methodology followed by a presentation of state-of-the-art methods. Content includes: - The foundations of Riemannian geometric methods for statistics on manifolds with emphasis on concepts rather than on proofs - Applications of statistics on manifolds and shape spaces in medical image computing - Diffeomorphic deformations and their applications As the methods described apply to domains such as signal processing (radar signal processing and brain computer interaction), computer vision (object and face recognition), and other domains where statistics of geometric features appear, this book is suitable for researchers and graduate students in medical imaging, engineering and computer science. - A complete reference covering both the foundations and state-of-the-art methods - Edited and authored by leading researchers in the field - Contains theory, examples, applications, and algorithms - Gives an overview of current research challenges and future applications

In the last twenty years, sub-Riemannian geometry has emerged as an independent research domain, with extremely rich motivations and ramifications in several parts of pure and applied mathematics, such as geometric analysis, geometric measure theory, stochastic calculus and evolution equations together with applications in mechanics, optimal control and biology. The aim of the lectures collected here is to present sub-Riemannian structures for the use of both researchers and graduate students.

This established reference work continues to lead its readers to some of the hottest topics of contemporary mathematical research. This new edition introduces and explains the ideas of the parabolic methods that have recently found such spectacular success in the work of Perelman at the examples of closed geodesics and harmonic forms. It also discusses further examples of geometric variational problems from quantum field theory, another source of profound new ideas and methods in geometry.

The heat kernel has long been an essential tool in both classical and modern mathematics but has become especially important in geometric analysis as a result of major innovations beginning in the 1970s. The methods based on heat kernels have been used in areas as diverse as analysis, geometry, and probability, as well as in physics. This book is a comprehensive introduction to heat kernel techniques in the setting of Riemannian manifolds, which inevitably involves analysis of the Laplace-Beltrami operator and the associated heat equation. The first ten chapters cover the foundations of the subject, while later chapters deal with more advanced results involving the heat kernel in a variety of settings. The exposition starts with an elementary introduction to Riemannian geometry, proceeds with a thorough study of the spectral-theoretic, Markovian, and smoothness properties of the Laplace and heat equations on Riemannian manifolds, and concludes with Gaussian estimates of heat kernels. Grigor'yan has written this book with the student in mind, in particular by including over 400 exercises. The text will serve as a bridge between basic results and current research.

This volume contains a collection of well-written surveys provided by experts in Global Differential Geometry to give an overview over recent developments in Riemannian Geometry, Geometric Analysis and Symplectic Geometry. The papers are written for graduate students and researchers with a general interest in geometry, who want to get acquainted with the current trends in these central fields of modern mathematics.

Differential Geometry is a wide field. We have chosen to concentrate upon certain aspects that are appropriate for an introduction to the subject; we have not attempted an encyclopedic treatment. Book III is aimed at the first-year graduate level but is certainly accessible to advanced undergraduates. It deals with invariance theory and discusses invariants both of Weyl and not of Weyl type; the Chern?Gauss?Bonnet formula is treated from this point of view. Homothety homogeneity, local homogeneity, stability theorems, and Walker geometry are discussed. Ricci solitons are presented in the contexts of Riemannian, Lorentzian, and affine geometry.

This book presents a selection of the most recent algorithmic advances in Riemannian geometry in the context of machine learning, statistics, optimization, computer vision, and related fields. The unifying theme of the different chapters in the book is the exploitation of the geometry of data using the mathematical machinery of Riemannian geometry. As demonstrated by all the chapters in the book, when the data is intrinsically non-Euclidean, the utilization of this geometrical information can lead to better algorithms that can capture more accurately the structures inherent in the data, leading ultimately to better empirical performance. This book is not intended to be an encyclopedic compilation of the applications of Riemannian geometry. Instead, it focuses on several important research directions that are currently actively pursued by researchers in the field. These include statistical modeling and analysis on manifolds,optimization on manifolds, Riemannian manifolds and kernel methods, and dictionary learning and sparse coding on manifolds. Examples of applications include novel algorithms for Monte Carlo sampling and Gaussian Mixture Model fitting, 3D brain image analysis,image classification, action recognition, and motion tracking.

Elie Cartan's book Geometry of Riemannian Manifolds (1928) was one of the best introductions to his methods. It was based on lectures given by the author at the Sorbonne in the academic year 1925-26. A modernized and extensively augmented edition appeared in 1946 (2nd printing, 1951, and 3rd printing, 1988). Cartan's lectures in 1926-27 were different -- he introduced exterior forms at the very beginning and used extensively orthonormal frames throughout to investigate the geometry of Riemannian manifolds. In this course he solved a series of problems in Euclidean and non-Euclidean spaces, as well as a series of variational problems on geodesics. The lectures were translated into Russian in the book Riemannian Geometry in an Orthogonal Frame (1960). This book has many innovations, such as the notion of intrinsic normal differentiation and the Gaussian torsion of a submanifold in a Euclidean multidimensional space or in a space of constant curvature, an affine connection defined in a normal fiber bundle of a submanifold, etc. The only book of Elie Cartan that was not available in English, it has now been translated into English by Vladislav V Goldberg, the editor of the Russian edition.

The present book contains the lecture notes from a "Nachdiplomvorlesung", a topics course adressed to Ph. D. students, at the ETH ZUrich during the winter term 95/96. Consequently, these notes are arranged according to the requirements of organizing the material for oral exposition, and the level of difficulty and the exposition were adjusted to the audience in Zurich. The aim of the course was to introduce some geometric and analytic concepts that have been found useful in advancing our understanding of spaces of nonpos itive curvature. In particular in recent years, it has been realized that often it is useful for a systematic understanding not to restrict the attention to Riemannian manifolds only, but to consider more general classes of metric spaces of generalized nonpositive curvature. The basic idea is to isolate a property that on one hand can be formulated solely in terms of the distance function and on the other hand is characteristic of nonpositive sectional curvature on a Riemannian manifold, and then to take this property as an axiom for defining a metric space of nonposi tive curvature. Such constructions have been put forward by Wald, Alexandrov, Busemann, and others, and they will be systematically explored in Chapter 2. Our focus and treatment will often be different from the existing literature. In the first Chapter, we consider several classes of examples of Riemannian manifolds of nonpositive curvature, and we explain how conditions about nonpos itivity or negativity of curvature can be exploited in various geometric contexts.

This is a textbook for Riemannian Geometry and Geometric Analysis, introducing techniques from nonlinear analysis at an early

stage. Such techniques have recently become an indispensible tool in research in geometry and they are treated in a textbook for the first time. Subjects treated are: Differentiable and Riemannian manifolds, metric properties, tensor calculus, vector bundles, the Hodge theorem for de Rham cohomology, connections and curvature, the Yang-Mills functional, geodesics and Jacobi fields, Rauch comparison theorem and applications, Morse theory (including an introduction to algebraic topology), applications to the existence of closed geodesics, symmetric spaces and Kahler manifolds, the Palais-Smale condition and closed geodesics, harmonic maps, definition and basic properties, existence and uniqueness theorems, applications, minimal surfaces, regularity results. In an appendix Sobolev spaces and regularity theory for linear elliptic equations are discussed in detail. The book gives both invariant global notation and tensor notation and should thus prove to be of interest to physicists as well.
Book IV continues the discussion begun in the first three volumes. Although it is aimed at first-year graduate students, it is also intended to serve as a basic reference for people working in affine differential geometry. It also should be accessible to undergraduates interested in affine differential geometry. We are primarily concerned with the study of affine surfaces which are locally homogeneous. We discuss affine gradient Ricci solitons, affine Killing vector fields, and geodesic completeness. Opozda has classified the affine surface geometries which are locally homogeneous; we follow her classification. Up to isomorphism, there are two simply connected Lie groups of dimension 2. The translation group R2 is Abelian and the ax + b group is non-Abelian. The first chapter presents foundational material. The second chapter deals with Type A surfaces. These are the left-invariant affine geometries on R2. Associating to each Type A surface the space of solutions to the quasi-Einstein equation corresponding to the eigenvalue ? = -1 turns out to be a very powerful technique and plays a central role in our study as it links an analytic invariant with the underlying geometry of the surface. The third chapter deals with Type B surfaces; these are the left-invariant affine geometries on the ax + b group. These geometries form a very rich family which is only partially understood. The only remaining homogeneous geometry is that of the sphere S2. The fourth chapter presents relations between the geometry of an affine surface and the geometry of the cotangent bundle equipped with the neutral signature metric of the modified Riemannian extension.
This book introduces readers to the living topics of Riemannian Geometry and details the main results known to date. The results are stated without detailed proofs but the main ideas involved are described, affording the reader a sweeping panoramic view of almost the entirety of the field. From the reviews "The book has intrinsic value for a student as well as for an experienced geometer. Additionally, it is really a compendium in Riemannian Geometry." --MATHEMATICAL REVIEWS
This book is based on the experience of teaching the subject by the author in Russia, France, South Africa and Sweden. The author provides students and teachers with an easy to follow textbook spanning a variety of topics on tensors, Riemannian geometry and geometric approach to partial differential equations. Application of approximate transformation groups to the equations of general relativity in the de Sitter space simplifies the subject significantly.
This book focuses on Hamilton's Ricci flow, beginning with a detailed discussion of the required aspects of differential geometry, progressing through existence and regularity theory, compactness theorems for Riemannian manifolds, and Perelman's noncollapsing results, and culminating in a detailed analysis of the evolution of curvature, where recent breakthroughs of Böhm and Wilking and Brendle and Schoen have led to a proof of the differentiable 1/4-pinching sphere theorem.
* A geometric approach to problems in physics, many of which cannot be solved by any other methods * Text is enriched with good examples and exercises at the end of every chapter * Fine for a course or seminar directed at grad and adv. undergrad students interested in elliptic and hyperbolic differential equations, differential geometry, calculus of variations, quantum mechanics, and physics
This book covers the topics of differential manifolds, Riemannian metrics, connections, geodesics and curvature, with special emphasis on the intrinsic features of the subject. It treats in detail classical results on the relations between curvature and topology. The book features numerous exercises with full solutions and a series of detailed examples are picked up repeatedly to illustrate each new definition or property introduced.